

On possible implications of gluon number fluctuations in DIS data

Misha Kozlov, Arif Shoshi and Wenchang Xiang

*Fakultät für Physik, Universität Bielefeld,
D-33501 Bielefeld, Germany*

E-mail: mkozlov@physik.uni-bielefeld.de, shoshi@physik.uni-bielefeld.de,
wcxiang@physik.uni-bielefeld.de

ABSTRACT: We study the effect of gluon number fluctuations (Pomeron loops) on deep inelastic scattering (DIS) in the fixed coupling case. We find that the description of the DIS data is improved once gluon number fluctuations are included. Also the values of the parameters, like the saturation exponent and the diffusion coefficient, turn out reasonable and agree with values obtained from numerical simulations of toy models which take into account fluctuations. This outcome seems to indicate the evidence of geometric scaling violations, and a possible implication of gluon number fluctuations, in the DIS data. However, we cannot exclude the possibility that the scaling violations may also come from the diffusion part of the solution to the BK-equation, instead of gluon number fluctuations.

KEYWORDS: Deep Inelastic Scattering, Asymptotic Freedom.

Contents

1.	Introduction	1
2.	Mean field approximation	3
3.	Beyond the mean field approximation	4
3.1	Event-by-event scattering amplitude	4
3.2	Physical scattering amplitude	5
4.	Numerical results	6

1. Introduction

The mean-field dynamics of the high-energy dipole-proton scattering is described by the BK-equation [1]. Phenomenological ansätze for the dipole-proton scattering amplitude $T(r, x)$ (where r is the transverse dipole size and x the Bjorken-variable) inspired by the BK-equation have led to quite successful descriptions of the HERA data. The T -matrix following from the BK-equation shows within a restricted kinematical window, which increases with collision energy, the geometric scaling behaviour [2–4], $T(r, x) = T(r^2 Q_s^2(x))$, where $Q_s(x)$ is the saturation scale, which seems well supported by the HERA data [5]. The correction to the solution outside the restricted window, the “BK-diffusion term”, violates the geometric scaling [2–4] and depends on the variable $\ln(1/r^2 Q_s^2(x))/\sqrt{D_{\text{BK}} Y}$. Iancu, Itakura and Munier (IIM) [6] have shown that the “BK-diffusion term”, giving a substantial amount of geometric scaling violations, is needed in order to accurately describe the experimental HERA data. The exponent λ of the saturation scale, $Q_s^2(x) \simeq (x_0/x)^\lambda$, is known at NLO [7], $\lambda \simeq 0.3$, and agrees with the values extracted from fits to HERA data.

Recently, there has been a tremendous theoretical progress in understanding the high-energy QCD evolution beyond the mean field approximation, i.e. beyond the BK-equation. It has been understood how to include discreteness and fluctuations of gluon numbers (Pomeron loops) in small- x evolution [8–12]. After including these elements, the evolution becomes stochastic and one has to distinguish between the event-by-event amplitude $T(r, x)$, which corresponds to an individual gluon number realization, and the physical amplitude $\langle T(r, Y) \rangle$, which one obtains by averaging over all individual realizations [9]. At very high energy, the discreteness effect *decreases* the exponent λ as compared to BK-value and the gluon number fluctuations, i.e., the averaging over all events to calculate the physical amplitude, replaces the geometric scaling resulting from the BK-equation (in the “wave front” region) by a new scaling [8, 9], the *diffusive scaling*, namely, $\langle T(r, Y) \rangle$ is a function of a single variable $\ln(1/r^2 Q_s^2(x))/\sqrt{D Y}$, where D is the diffusion coefficient. The value

of D determines the rapidity above which gluon number fluctuations become important, $Y \geq Y_D = 1/D$, which is the case when the fluctuation of the saturation scales of the individual events becomes large, in formulas, when the dispersion $\sigma^2 = 2(\langle \rho_s^2(Y) \rangle - \langle \rho_s(Y) \rangle^2) = DY \gg 1$, where $\rho_s(Y) = \ln(Q_s^2(Y)/Q_0^2)$. At high energy, such that $\sigma^2 \gg 1$, it has been shown that fluctuations do strongly modify measurable quantities [13, 14]. (A more detailed presentation of the recent theoretical progress is given in refs. [15] while the most recent studies on Pomeron loops based on toy models can be found in refs. [16–24].)

In this work we elaborate, in a quite approximative way, whether the HERA data [25] do indicate any possible implication of gluon number fluctuations. The coupling is kept fixed throughout this work. We proceed in the following way: We use for the event-by-event amplitude several models, the GBW model [26], the IIM model [6] and a model which is close to the theoretical findings for T at very large energy (see eq. (3.2)). For the averaging over all events we use the high-energy QCD/statistical physics correspondence [9], i.e., a Gaussian for the distribution of $\rho_s(Y) = \ln(Q_s^2(Y)/Q_0^2)$. Moreover, assuming that the DIS cross section shows diffusive scaling in the HERA energy range, we have used the “quality factor” method of ref. [27] to get an estimation for the value of λ , in a model-independent way. The procedure we use in this work is always based on approximations and, therefore, can at best give hints on a possible implication of gluon number fluctuations in the HERA data.

After including fluctuations in the way described above, we obtain from the analysis of the HERA data values for the exponent λ and the diffusion coefficient D which are quite independent of the ansatz for the event-by-event amplitude. Also the model-independent approach gives a similar value for λ . We find $\lambda \simeq 0.2$ which is smaller than the value from the BK-inspired models (no fluctuations), $\lambda \simeq 0.3$, and the decrease is in agreement with theoretical expectations. For the diffusion coefficient we find a sizeable value, $D \simeq 0.35$. Surprisingly, this value is very close to the values found for D in numerical simulation of the $(1+1)$ dimensional model [19] and of evolution equations in QCD [28] (approximations to Pomeron loop equations [10–12]) in the fixed coupling case. The sizeable value of D may indicate a possible involvement of fluctuations in the HERA data since $Y \geq Y_D = 1/D$ for rapidities at HERA.

We observe that after including fluctuations the description of the HERA data is improved for all models we have used for the event-by-event amplitude. In the case of the GBW model, which exhibits pure geometric scaling, after the inclusion of fluctuations, which lead to a violation of geometric scaling, a much better description is obtained, namely, $\chi^2/\text{d.o.f} = 1.74$ without and $\chi^2/\text{d.o.f} = 1.14$ with fluctuations. The situation seems to be similar with all event-by-event amplitudes which show geometric scaling. In the case of the IIM model, which contains already the geometric scaling violating BK-diffusion term, the inclusion of fluctuations also improves, however less than in the GBW case, the description of the HERA data; $\chi^2/\text{d.o.f} = 0.983$ before and $\chi^2/\text{d.o.f} = 0.807$ after including fluctuations.¹ The outcomes seem to tell us that violations of geometric scaling are required for an accurate description of the HERA data. The improvement of the

¹The χ^2 is defined such that the smallest χ^2 gives the best description to the HERA data.

description of the HERA data together with the very reasonable values for the parameters discussed above seem to indicate that gluon number fluctuations may be the reason for geometric scaling violations in the HERA data. However, we wish to emphasize here that the BK-diffusion term gives similar geometric scaling violations as fluctuations and may as well be the reason for the geometric scaling violations in the HERA data.

This work is organized as follows: In section 2, we show the results for the T -matrix for dipole-proton scattering and for the energy dependence of the saturation scale which are obtained in the mean field approximations, i.e., from the BK-equation. The results for the same quantities beyond the mean field approximation, or the effects of discreteness and fluctuations in gluon numbers on these quantities, are summarized in section 3. Finally, we give numerical results and discuss a possible implication of the physics beyond the mean field approximation in the HERA data.

2. Mean field approximation

In the mean field approximation, the Y -dependence of the T -matrix for a dipole of transverse size r scattering off a proton is given by the BK-equation. In the fixed coupling case, the solution to the BK-equation in the saturation region, where $T \simeq 1$, is [29]

$$T(r, Y) = 1 - C_0 \exp[-C_1(\rho - \rho_s(Y))^2] \quad \text{for } \rho - \rho_s(Y) \ll 1, \quad (2.1)$$

while for the front of the T -matrix, where $T \ll 1$ (but not too small), one finds [2, 4]

$$T(r, Y) = C_2 [\rho - \rho_s(Y) + C_3] \exp\left[-\lambda_s(\rho - \rho_s(Y)) - \frac{(\rho - \rho_s(Y))^2}{2\bar{\alpha}\chi''(\lambda_s)Y}\right] \\ \text{for } 1 \ll \rho - \rho_s(Y) \ll 2\chi''(\lambda_s)\bar{\alpha}_s Y, \quad (2.2)$$

where we have used $\bar{\alpha}_s = \alpha_s N_c / \pi$, $\rho = \ln(1/r^2 Q_0^2)$ and $\rho_s(Y) = \ln(Q_s^2(Y)/Q_0^2)$ with $Q_s(Y)$ the saturation scale. In above equations, the constants C_0, C_2, C_3 are of $\mathcal{O}(1)$, $C_1 = -C_F(1 - \lambda_0)/N_c 2\chi(\lambda_s)$ (C_F is the casimir factor in the fundamental representation), $\lambda_s = 0.6275$, and $\chi(\lambda) = 2\psi(1) - \psi(\lambda) - \psi(1 - \lambda)$ is the eigenvalue of the BFKL kernel. For the rapidity dependence of the saturation scale, which separates the saturated ($r \gg 1/Q_s(Y)$) from the dilute ($r \ll 1/Q_s(Y)$) regime, one obtains from the BK equation [2, 4]

$$Q_s^2(Y) = Q_0^2 \frac{\exp[\bar{\alpha}\chi'(\lambda_s)Y]}{[\bar{\alpha}Y]^{\frac{3}{2(1-\lambda_0)}}}. \quad (2.3)$$

Note that within the even more restricted window, $\rho - \rho_s(Y) \ll \sqrt{2\chi''(\lambda_s)\bar{\alpha}_s Y}$, where the diffusion term in the exponent in eq. (2.2) can be neglected, the T -matrix shows the geometric scaling behaviour, i.e., it depends only on the difference $\rho - \rho_s(Y)$ instead of depending on r and Y separately. At very large r , so that $\rho - \rho_s(Y) \gg 2\chi''(\lambda_s)\bar{\alpha}_s Y$, the T -matrix exhibits color transparency, i.e., it shows a faster decrease with ρ as compared to eq. (2.2); $T \sim \exp[-\rho]$.

Iancu, Itakura and Munier [6] have used the following ansatz for the T -matrix,

$$T^{\text{IIM}}(r, Y) = \begin{cases} 1 - \exp[-a \ln^2(b r Q_s(x))] & , \quad r Q_s(x) > 2 \\ N_0 \left(\frac{r Q_s(x)}{2}\right)^{2\left(\lambda_s + \frac{\ln(2/r Q_s(x))}{\kappa \lambda Y}\right)} & , \quad r Q_s(x) < 2, \end{cases} \quad (2.4)$$

which obviously includes the features of the solution to the BK equation, to compare the theory in the mean field approximation with the DIS data. They have used for the saturation momentum the leading Y -dependence of eq. (2.3), $Q_s(x) = (x_0/x)^\lambda$, however, with λ and x_0 being fixed by fitting the DIS data. The constant $\kappa = \chi''(\lambda_s)/\chi'(\lambda_s) \approx 9.9$ is a LO result coming from the BK-equation, N_0 is a constant around 0.5 and a and b are determined by matching the two pieces in eq. (2.4) at $r Q_s = 2$.

The ‘‘BK-diffusion term’’ in the IIM-ansatz (2.4),

$$\left(\frac{r Q_s(x)}{2}\right)^{2\frac{\ln(2/r Q_s(x))}{\kappa \lambda Y}} = \exp\left[-\frac{\ln^2(4/r^2 Q_s^2(x))}{2 \kappa \lambda Y}\right], \quad (2.5)$$

which is the quadratic term in the exponent of eq. (2.2), does explicitly violate the geometric scaling behaviour. We wish to emphasize here that, as also shown in [6], this violation seems required in order to get an accurate description of the DIS data. Without it, even allowing λ_s to be an additional fitting parameter, one can not get a better description of the DIS data. For further details on the importance of the diffusion term see ref. [6].

In this work, we wish to elaborate whether the violation of the geometric scaling may come from gluon number fluctuations (Pomeron loops) and not from the BK-equation. As we will see in the next sections, the fluctuations do indeed give a similar violation of the geometric scaling and also lead to a better description of the DIS data as compared to the case where the T -matrix shows a geometric scaling behaviour.

3. Beyond the mean field approximation

To go beyond the mean field approximation one has to include the effect of discreteness and fluctuations of gluon numbers [8, 9]. After including fluctuations one has to distinguish between the even-by-event amplitude and the averaged (physical) amplitude. They can be explained by considering the evolution of a proton from $y = 0$ up to $y = Y$ which is probed by a dipole of size r , giving the amplitude $\bar{T}(r, Y)$. The evolution of the proton is stochastic and leads to random gluon number realizations inside the proton at Y , corresponding to different events in an experiment. The physical amplitude, $\bar{T}(r, Y)$, is then given by averaging over all possible gluon number realizations/events, $\bar{T}(r, Y) = \langle T(r, Y) \rangle$, where $T(r, Y)$ is the amplitude for the dipole r scattering off a particular realization of the evolved proton at Y . In the following we discuss the event-by-event amplitude $T(r, Y)$ and the averaged amplitude $\bar{T}(r, Y)$.

3.1 Event-by-event scattering amplitude

In a single scattering process, the mean field approximation breaks down when the occupancy of gluons inside the evolved proton is low so that the discreteness of the gluon

number needs to be taken into account; the number of gluons cannot be non-zero and less than one since it has to be discrete. When including the discreteness effect, as compared to the results from the BK-equation, the energy dependence of the saturation momentum changes to [8, 9]

$$Q_s^2(Y) = Q_0^2 \exp \left[\bar{\alpha}_s \chi'(\lambda_s) Y \left(1 - \frac{\pi^2 \chi''(\lambda_s)}{2(\Delta\rho)^2 \chi(\lambda_s)} \right) \right] \quad (3.1)$$

and the piecewise, approximate, shape of the T -matrix at fixed coupling and very high energy reads [8, 9]

$$T(r, Y) = \begin{cases} 1 & \text{for } \rho - \rho_s(Y) \ll 0 \\ N_1 [\rho - \rho_s(Y)] e^{\lambda_s [\rho - \rho_s(Y)]} & \text{for } 0 < \rho - \rho_s(Y) < \Delta\rho \\ N_2 e^{-[\rho - \rho_s(Y)]} & \text{for } \rho - \rho_s(Y) \gg \Delta\rho \end{cases} \quad (3.2)$$

where N_1 and N_2 are irrelevant constants and the front width is $\Delta\rho \simeq (1/\lambda_s) \ln(1/\alpha^2)$. The front width cannot be larger than $\Delta\rho$ which is the distance when the amplitude decreases from its maximal value $T \approx 1$ down to the value $T = \mathcal{O}(\alpha^2)$ where the discreteness of gluon numbers becomes important. The width is formed via diffusion, $\rho - \rho_s(Y) \propto \sqrt{\alpha Y}$, and it requires the rapidity $Y_F \simeq (\Delta\rho)^2 / (2\chi''(\lambda_s)\alpha)$ until it is completed. The event-by-event amplitude given in eq. (3.2), which is formed at $Y > Y_F$, shows, approximately, geometric scaling: $T(r, Y) \approx T(\rho - \rho_s(Y))$.

The main differences as compared to the mean-field results are: The exponent of the saturation scale in the event-by-event amplitude, cf. eq. (3.1) and eq. (2.3), is decreased due to the discreteness of gluon numbers. Further the width of the front of the event-by-event amplitude is fixed, $\Delta\rho$, instead of increasing with rapidity as in eq. (2.2).

3.2 Physical scattering amplitude

Based on the relation between high-energy QCD evolution and reaction-diffusion processes in statistical physics [9], the fluctuations in gluon numbers are taken into account by averaging over all event-by-event amplitudes,

$$\langle T((\rho - \rho_s(Y))) \rangle = \int d\rho_s T(\rho - \rho_s(Y)) P(\rho_s(Y) - \langle \rho_s(Y) \rangle) , \quad (3.3)$$

where the distribution of $\rho_s(Y)$ is, to a very good approximation, a Gaussian [31]:

$$P(\rho_s) \simeq \frac{1}{\sqrt{\pi\sigma^2}} \exp \left[-\frac{(\rho_s - \langle \rho_s \rangle)^2}{\sigma^2} \right] . \quad (3.4)$$

The expectation value of the front position, $\langle \rho_s(Y) \rangle$, increases with rapidity as $\langle \rho_s(Y) \rangle = \ln(Q_s^2(Y)/Q_0^2)$ at high energy [9], with $Q_s(Y)$ given in eq. (3.1). The dispersion of the front at high energy increases linearly with rapidity,

$$\sigma^2 = 2 [\langle \rho_s^2 \rangle - \langle \rho_s \rangle^2] = D Y \quad (3.5)$$

where D is the diffusion coefficient, whose value is known only for $\alpha \rightarrow 0$ (asymptotic energy) [32, 8]. Since the values of D and the exponent λ of the saturation scale, $Q_s^2(x) = 1 \text{ GeV}^2 (x_0/x)^\lambda$, see eq. (3.1), are not known for finite energies, e.g. at HERA energy, in what follows we will treat them as free parameters.

At very high energy, such that $\sigma^2 \gg 1$, the dispersion of the fronts due to the gluon number fluctuations from event to event has large consequences on $\langle T(r, Y) \rangle$: the geometric scaling of the single events $T(\rho - \rho_s(Y))$, cf. eq. (3.2), is replaced by a new form of scaling, known as diffusive scaling, namely, $\langle T(r, Y) \rangle$ is a function of $(\rho - \langle \rho_s(Y) \rangle) / \sqrt{DY}$,

$$\langle T(r, Y) \rangle = \bar{T}(r, Y) = \bar{T} \left(\frac{\rho - \langle \rho_s(Y) \rangle}{\sqrt{DY}} \right). \quad (3.6)$$

The diffusive scaling is expected to set in at $Y > Y_D = 1/D$, which follows from the requirement $\sigma^2 \gg 1$.

The goal of this work is to study whether the diffusive scaling behaviour of the dipole-proton scattering amplitude in eq. (3.6), which is caused by gluon number fluctuations, may be present in the HERA data. As we will see in the next section, the fluctuations do improve the description of the HERA data, indicating that the violation of geometric scaling seems important for an accurate description of the data. We will discuss whether the violation preferred by the DIS data is due to the gluon number fluctuations, which lead to the diffusive scaling $(\rho - \rho_s(Y)) / \sqrt{DY}$, or due to the BK diffusion term, cf. eq. (2.5), which corrects the geometric scaling in a similar way, namely, via $(\rho - \rho_s(Y)) / \sqrt{2\bar{\alpha}\chi^n Y}$.

4. Numerical results

Our fit includes the ZEUS data for the F_2 structure function,

$$\begin{aligned} F_2(x, Q^2) &= \frac{Q^2}{4\pi^2\alpha_{em}} (\sigma_T(x, Q^2) + \sigma_L(x, Q^2)), \\ \sigma_{T,L}(x, Q^2) &= \int dz d^2r |\psi_{T,L}(z, r, Q^2)|^2 \sigma_{dip}(x, r) \end{aligned} \quad (4.1)$$

in the kinematical range $x \leq 10^{-2}$ and $0.045 \text{ GeV}^2 < Q^2 < 50 \text{ GeV}^2$ (see also [6] for more discussions on the range). The upper limit on Q^2 has been chosen large enough to include a large amount of “perturbative” data points, but low enough in order to justify the use of the BFKL dynamics, rather than DGLAP evolution. We use in our fit the same photon wave functions $\psi_{T,L}$ as in ref. [26], which are computable in QED, and three quarks with equal mass, $m_q = 140 \text{ MeV}$. We have considered only the ZEUS data because there is a mismatch between the H1 and ZEUS with regard to the data normalization and since only ZEUS has data also in the low Q^2 region, i.e., in the saturation region. To fix the parameters we minimize $\chi^2 = \sum_i (\text{model}(i, p_1, \dots, p_n) - F_2(i))^2 / (\text{error}(i))^2$, where the sum goes over the data points, p_1, \dots, p_n denote the parameters to be found, $F_2(i)$ the experimental results for the F_2 structure function, and for the error of F_2 , i.e., $(\text{error}(i))^2$, we use the systematic error squared plus the statistical error squared.

The interesting ingredient for us in eq. (4.1) is the dipole-proton cross section, $\sigma_{dip} = 2\pi R^2 \langle T(r, x) \rangle$, with $2\pi R^2$ being the outcome of the integration over the impact parameter.

We will use different ansätze for the event-by-event amplitude, $T(r, x)$, and the physical amplitude, $\langle T(r, x) \rangle$, is obtained according to the rules outlined in section 3.2. (We wish to note that the ansätze for $T(r, x)$, which are derived/motivated based on perturbative QCD, are used to describe also the low virtuality data, $Q^2 \leq 1 \text{ GeV}^2$, in the fit to the HERA data. In this region non-perturbative physics [33] is involved which is only approximately given by our ansätze.) In σ_{dip} we will use the event-by-event amplitude and the physical amplitude in order to study the effects of gluon number fluctuations. In the case of $T(r, x)$ there are three free parameters which will be fixed by fitting the HERA data: R (“radius of the proton”) and x_0 and λ coming via the saturation momentum $Q_s^2(x) = 1 \text{ GeV}^2 (x_0/x)^\lambda$. In the case of the averaged (physical) amplitude, $\langle T(r, x) \rangle$, there is another free parameter, the diffusion coefficient D .

- Golec-Biernat, Wüsthoff (GBW) model [26]:

The GBW model

$$T^{GBW}(r, x) = 1 - \exp \left[-\frac{r^2 Q_s^2(x)}{4} \right], \quad (4.2)$$

is one of the most simple models which shows geometric scaling, $T(r, x) = T(r^2 Q_s^2(x))$, and leads to a quite successful description of the HERA data, as can be seen from figures 1, 2 and the χ^2 (error) in table 1 (denoted by GBW). It is nice to see that the value of the saturation exponent, $\lambda \simeq 0.285$, which is found by fitting the HERA data with the GBW model, comes out close to the theoretical NLO results for λ [7].

Now, using the GBW model as an event-by-event amplitude, we include the effect of gluon number fluctuations by averaging over all events via eq. (3.3). The resulting $\langle T^{GBW}(r, x) \rangle$, which breaks the geometric scaling, leads to a relatively much better description of the HERA data, as can be seen from the comparison of the χ^2 values and the two lines in figures 1, 2. The large improvement after including fluctuations seems to indicate that violations of geometric scaling, and probably even gluon number fluctuations, are implicated in the HERA data.

It is important to note that the values of the fitting parameters come out reasonable also after including the gluon number fluctuations. The value of λ becomes smaller after including fluctuations which is in agreement with theoretical expectations, as can be seen from the comparison of eq. (2.3) with eq. (3.1). Furthermore, the value of the diffusion coefficient D is sizeable, and is surprisingly close to the values which have been found numerically by solving the $(1+1)$ dimensional toy model [19] and the approximate QCD evolution equations [28] (they represent an approximation of the Pomeron loop equations [10–12]) in the fixed coupling case. Note also that the radius of the proton, R , increases somewhat and x_0 becomes smaller, meaning that $Q_s < 1 \text{ GeV}$ up to $x \simeq 10^{-6}$, due to fluctuations. Also the reasonable values of the parameters, especially the sizeable value of D yielding $Y_D = 1/D \simeq 2.5$, in addition to the better description of the HERA data after including fluctuations, seem to be in favour of an implication of gluon number fluctuations in the HERA data.

model/parameters	χ^2	$\chi^2/\text{d.o.f}$	$x_0 (\times 10^{-4})$	λ	$R(\text{fm})$	D
T^{GBW}	266.22	1.74	4.11	0.285	0.594	0
$\langle T^{\text{GBW}} \rangle$	173.39	1.14	0.0546	0.225	0.712	0.397

Table 1: GBW model: The parameters of the event-by-event (2 line) and of the physical (3 line) amplitude.

model/parameters	χ^2	$\chi^2/\text{d.o.f}$	$x_0 (\times 10^{-4})$	λ	$R(\text{fm})$	D
T^{IIM}	150.45	0.983	0.5379	0.252	0.709	0
$\langle T^{\text{IIM}} \rangle$	122.62	0.807	0.0095	0.198	0.812	0.325

Table 2: IIM model: The parameters of the event-by-event (2 line) and of the physical (3 line) amplitude.

- Iancu, Itakura, Munier (IIM) model [6], other models and a model-independent approach:

The IIM model given in eq. (2.4) includes the BK-diffusion term, $\ln(4/r^2 Q_s^2)/\sqrt{2\kappa\lambda Y}$, which explicitly violates the geometric scaling. It has been shown in [6] that this violation does noticeably improve the description of the HERA data in comparison with the GBW model, as can be seen from the much smaller χ^2 value in the IIM case in table 2 (we always use $N_0 = 0.5$ in the IIM model). In ref. [6] has been further shown that without the BK-diffusion term, although allowing for an additional free parameter λ_s (one parameter more than in the GBW model), the $\chi^2/\text{d.o.f}$ value does not improve and is close to the GBW value.

Note that the GBW model only after including gluon number fluctuations gives a $\chi^2/\text{d.o.f}$ value which is comparable with the IIM one. This may mean that the violation of the geometric scaling is favoured by the HERA data. The violation may come from the gluon number fluctuations or from the BK-diffusion term.

In the case of the IIM model, after including fluctuations, we can give an analytic expression for the physical amplitude

$$\begin{aligned}
 \langle T^{\text{IIM}}(r, Y) \rangle = & \frac{N_0}{2\sigma} \left[\sigma \text{Erfc} \left(\frac{\ln \frac{4}{r^2 Q_s^2}}{\sigma} \right) - \frac{\text{Exp} \left(\frac{-\frac{a}{4\sigma^2} \ln^2(b^2 r^2 Q_s^2)}{\frac{1}{\sigma^2} + \frac{a}{4}} \right)}{\sqrt{\frac{1}{\sigma^2} + \frac{a}{4}}} \right] \times \\
 & \times \text{Erfc} \left(\frac{\frac{a \ln(4b^2)}{4} + \frac{1}{\sigma^2} \ln \left(\frac{4}{r^2 Q_s^2} \right)}{\sqrt{\frac{1}{\sigma^2} + \frac{a}{4}}} \right) + \frac{1}{\sqrt{\frac{1}{2\kappa\lambda Y} + \frac{1}{\sigma^2}}} \left(1 + \text{Erf} \left(\frac{-\frac{\lambda_s}{2} + \frac{1}{\sigma^2} \ln \left(\frac{4}{r^2 Q_s^2} \right)}{\sqrt{\frac{1}{2\kappa\lambda Y} + \frac{1}{\sigma^2}}} \right) \right) \times \\
 & \times \text{Exp} \left(-\frac{\left[\frac{\ln^2 \left(\frac{4}{r^2 Q_s^2} \right)}{2\kappa\lambda Y \sigma^2} - \frac{\lambda_s^2}{4} + \lambda_s \frac{\ln \left(\frac{4}{r^2 Q_s^2} \right)}{\sigma^4} \right]}{\sqrt{\frac{1}{2\kappa\lambda Y} + \frac{1}{\sigma^2}}} \right) \Bigg],
 \end{aligned}$$

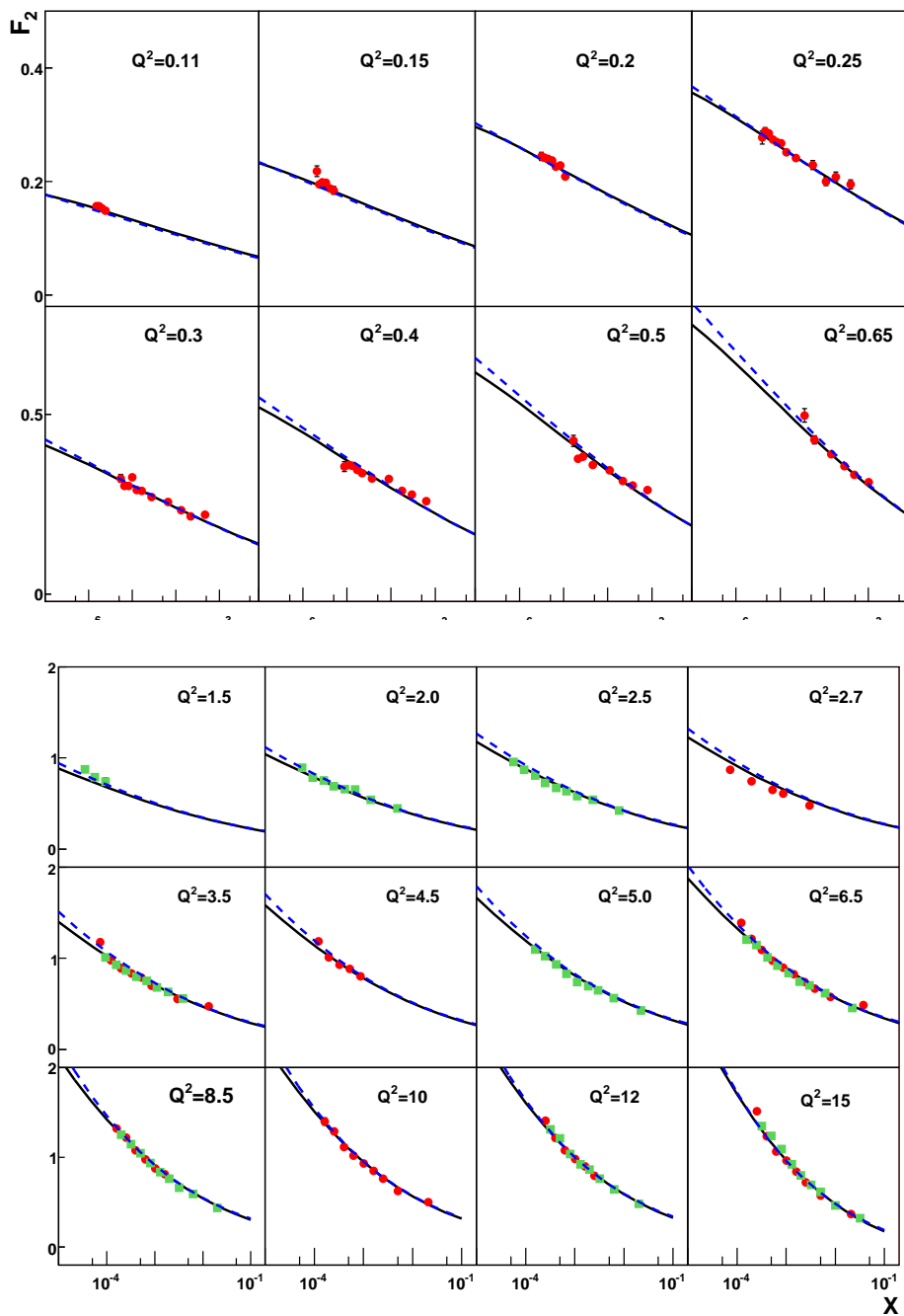


Figure 1: The F_2 structure function versus x at different values of Q^2 . The experimental points are the latest published data from the H1 (squares) and ZEUS (dots) collaborations [25]. The solid line represents the result of the averaged GBW fit and the dashed line the result of the GBW fit to the ZEUS data for $x \leq 10^{-2}$ and $0.045 \text{ GeV}^2 < Q^2 < 50 \text{ GeV}^2$. The data points at lowest Q^2 values, 0.045 , 0.065 and 0.085 GeV^2 , are not shown here although they are included in the fits.

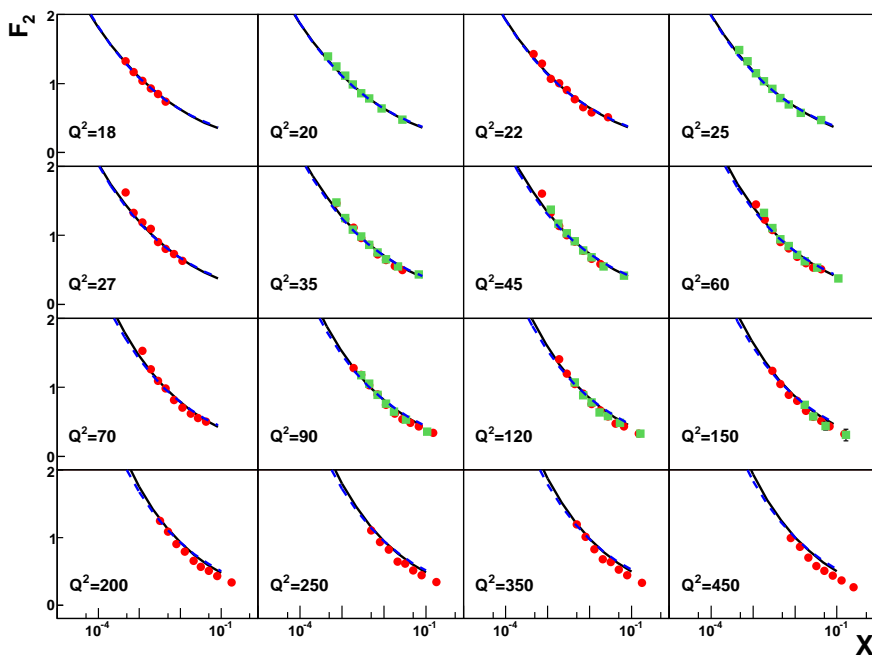


Figure 2: The same as in figure 1, but for larger values of Q^2 . Note that we show in this figure our results up the highest Q^2 although our fit is performed including only the data for $Q^2 < 50 \text{ GeV}^2$.

which can be used in phenomenological applications, where $\text{Erfc}(x)$ is the complementary error function. Also in the IIM case fluctuations do improve the description of the HERA data, however not much, as can be seen from the comparable $\chi^2/\text{d.o.f}$ values for T^{IIM} and $\langle T^{\text{IIM}} \rangle$ in table 2. This is so because the IIM model does already contain the geometric scaling violations via the BK-diffusion term, $\ln(4/r^2 Q_s^2)/\sqrt{2\kappa\lambda Y}$, and describes accurately the HERA data, before including fluctuations. However, note that the diffusion coefficients in case of fluctuations and the BK-diffusion term are quite different, namely, $D = 0.325$ and $2\kappa\lambda \simeq 3.9$, respectively.

After including fluctuations, the parameters in the GBW and the IIM case are close to each other. Apart from the fact that similar values for D are found in numerical simulations of evolution equations [28, 19] and the decrease of λ due to fluctuations is theoretically expected, at least at high energy, the parameters λ and D also seem to be quite model-independent. Indeed, similar values for λ and D would come out also if one uses a model² as suggested by the theoretical findings at high energy as given in eq. (3.2), for reasonable values of the proton radius, $R \simeq 0.7 - 0.8 \text{ fm}$. Moreover, the similar value of λ coming out of the different models is also supported by the following model-independent approach: In case fluctuation are important in the range of HERA

²Such a model would be for instance the IIM model with the diffusion variable $\ln(4/r^2 Q_s^2)/\sqrt{2\kappa\lambda Y}$ replaced by $\ln(4/r^2 Q_s^2)(1 - \lambda_s)/\sqrt{\Delta\rho}$, such that the new model interpolates between the three regions of eq. (3.2) and shows the geometric scaling behaviour. The constant $\Delta\rho$ is given by eq. (3.2). We use in $\Delta\rho$ a small value for α , $\alpha = 1/15$, which is the value required such that the exponent of Q_s^2 in eq. (2.3) agrees with experimental or NLO results, $\lambda \simeq 0.3$. With this input, we find for $R = 0.8 \text{ fm}$, the following results: $\lambda = 0.235$ and $D = 0.58$.

data, one finds the diffusive scaling behaviour [30], i.e., $\sigma^{\gamma^*p}/\sqrt{DY}$ is a function of $\ln(1/r^2 Q_s^2)/\sqrt{DY}$. Defining a “quality factor” $\mathcal{O}(\lambda, x_0, D)$ as done in [27], which test the quality of this diffusive scaling in HERA data, we find by minimizing it $\lambda = 0.215$, at least for the input-values $0.01 \leq D \leq 0.7$ which we have investigated.

The seemingly model-independent values of the parameters λ and D , their agreement with numerically found values, and the improvement of the description of the HERA data in all models after including fluctuations, seem to tell us that gluon number fluctuations are relevant in the range of HERA data. However, since in the case of the IIM model the fluctuations do not improve much the description of the HERA data, one may conclude that the BK-equation alone should describe the HERA data and that fluctuations are negligible in the energy range of the HERA data. The intention of this work is to illustrate the possibility that fluctuations may be present in the HERA data.

The situation would become more clear at even higher collision energies as compared to the HERA energy. With growing Y , according to the BK-equation the window for the geometric scaling behaviour would increase, and the scaling violating term would become less important. On the other side, the small- x dynamics including gluon number fluctuations leads to a more clear diffusive scaling behaviour with increasing Y . The forthcoming LHC may tell us more whether geometric or diffusive scaling is more appropriate for the description of the observables in the LHC energy range.

Throughout this work the coupling is kept fixed. As mentioned above, the $(1+1)$ dimensional model in [19], which accomodates Pomeron loops, gives similar values for D as our analysis for a fixed coupling. However, it has been recently shown that if allowing the coupling to run in this toy model [20] then the effects of gluon number fluctuations can be neglected up to energies far beyond the HERA and LHC energies. We plan to extend our work by the running coupling in order to see whether the HERA data can tell something about the running coupling and whether the prediction of the toy model remains valid also in the QCD case.

Acknowledgments

We are very thankful to Stephane Munier for providing us with the codes which he has used for the analysis of the HERA data with the IIM model. We are grateful to Edmond Iancu for valuable discussions. The authors acknowledge financial support by the Deutsche Forschungsgemeinschaft under contract Sh 92/2-1.

References

- [1] Y.V. Kovchegov, *Small- x F_2 structure function of a nucleus including multiple Pomeron exchanges*, *Phys. Rev. D* **60** (1999) 034008 [[hep-ph/9901281](#)];
I. Balitsky, *Operator expansion for high-energy scattering*, *Nucl. Phys. B* **463** (1996) 99 [[hep-ph/9509348](#)].
- [2] A.H. Mueller and D.N. Triantafyllopoulos, *The energy dependence of the saturation momentum*, *Nucl. Phys. B* **640** (2002) 331 [[hep-ph/0205167](#)].

- [3] E. Iancu, K. Itakura and L. McLerran, *Geometric scaling above the saturation scale*, *Nucl. Phys. A* **708** (2002) 327 [[hep-ph/0203137](#)].
- [4] S. Munier and R. Peschanski, *Geometric scaling as traveling waves*, *Phys. Rev. Lett.* **91** (2003) 232001 [[hep-ph/0309177](#)]; *Traveling wave fronts and the transition to saturation*, *Phys. Rev. D* **69** (2004) 034008 [[hep-ph/0310357](#)].
- [5] A.M. Stasto, K.J. Golec-Biernat and J. Kwiecinski, *Geometric scaling for the total $\gamma^* p$ cross-section in the low x region*, *Phys. Rev. Lett.* **86** (2001) 596 [[hep-ph/0007192](#)].
- [6] E. Iancu, K. Itakura and S. Munier, *Saturation and BFKL dynamics in the HERA data at small x* , *Phys. Lett. B* **590** (2004) 199 [[hep-ph/0310338](#)].
- [7] D.N. Triantafyllopoulos, *The energy dependence of the saturation momentum from RG improved BFKL evolution*, *Nucl. Phys. B* **648** (2003) 293 [[hep-ph/0209121](#)].
- [8] A.H. Mueller and A.I. Shoshi, *Small- x physics beyond the Kovchegov equation*, *Nucl. Phys. B* **692** (2004) 175 [[hep-ph/0402193](#)].
- [9] E. Iancu, A.H. Mueller and S. Munier, *Universal behavior of QCD amplitudes at high energy from general tools of statistical physics*, *Phys. Lett. B* **606** (2005) 342 [[hep-ph/0410018](#)].
- [10] A.H. Mueller, A.I. Shoshi and S.M.H. Wong, *Extension of the JIMWLK equation in the low gluon density region*, *Nucl. Phys. B* **715** (2005) 440 [[hep-ph/0501088](#)].
- [11] E. Iancu and D.N. Triantafyllopoulos, *A langevin equation for high energy evolution with Pomeron loops*, *Nucl. Phys. A* **756** (2005) 419 [[hep-ph/0411405](#)]; *Non-linear QCD evolution with improved triple-Pomeron vertices*, *Phys. Lett. B* **610** (2005) 253 [[hep-ph/0501193](#)].
- [12] A. Kovner and M. Lublinsky, *In pursuit of Pomeron loops: the JIMWLK equation and the Wess-Zumino term*, *Phys. Rev. D* **71** (2005) 085004 [[hep-ph/0501198](#)].
- [13] M. Kozlov, A.I. Shoshi and B.-W. Xiao, *Total gluon shadowing due to fluctuation effects*, *Nucl. Phys. A* **792** (2007) 170 [[hep-ph/0612053](#)]; *Fluctuation effects on R_{pA} at high energy*, [arXiv:0706.3998](#).
- [14] E. Iancu, C. Marquet and G. Soyez, *Forward gluon production in hadron hadron scattering with Pomeron loops*, *Nucl. Phys. A* **780** (2006) 52 [[hep-ph/0605174](#)].
- [15] A.H. Mueller, *Saturation and high density QCD*, [hep-ph/0501012](#); E. Iancu, *Stochastic aspects of QCD at high energy*, *Eur. Phys. J. C* **43** (2005) 345; S. Munier, *Cross-fertilization of QCD and statistical physics: high energy scattering, reaction-diffusion, selective evolution, spin glasses and their connections*, *Acta Phys. Polon. B* **37** (2006) 3451 [[hep-ph/0609161](#)]; D.N. Triantafyllopoulos, *Pomeron loops in high energy QCD*, *Acta Phys. Polon. B* **36** (2005) 3593 [[hep-ph/0511226](#)]; A.I. Shoshi, *High energy QCD beyond the mean field approximation*, [arXiv:0708.4316](#).
- [16] A.I. Shoshi and B.-W. Xiao, *Pomeron loops in zero transverse dimensions*, *Phys. Rev. D* **73** (2006) 094014 [[hep-ph/0512206](#)]; *Diffraction dissociation including Pomeron loops in zero transverse dimensions*, *Phys. Rev. D* **75** (2007) 054002 [[hep-ph/0605282](#)].
- [17] S. Bondarenko, L. Motyka, A.H. Mueller, A.I. Shoshi and B.W. Xiao, *On the equivalence of reggeon field theory in zero transverse dimensions and reaction-diffusion processes*, *Eur. Phys. J. C* **50** (2007) 593 [[hep-ph/0609213](#)].

- [18] J.P. Blaizot, E. Iancu and D.N. Triantafyllopoulos, *A zero-dimensional model for high-energy scattering in QCD*, *Nucl. Phys. A* **784** (2007) 227 [[hep-ph/0606253](#)].
- [19] E. Iancu, J.T. de Santana Amaral, G. Soyez and D.N. Triantafyllopoulos, *One-dimensional model for QCD at high energy*, *Nucl. Phys. A* **786** (2007) 131 [[hep-ph/0611105](#)].
- [20] A. Dumitru, E. Iancu, L. Portugal, G. Soyez and D.N. Triantafyllopoulos, *Pomeron loop and running coupling effects in high energy QCD evolution*, [arXiv:0706.2540](#).
- [21] S. Munier, *Dense-dilute factorization for a class of stochastic processes and for high energy QCD*, *Phys. Rev. D* **75** (2007) 034009 [[hep-ph/0608036](#)].
- [22] M. Kozlov and E. Levin, *Solution for the BFKL Pomeron calculus in zero transverse dimensions*, *Nucl. Phys. A* **779** (2006) 142 [[hep-ph/0604039](#)].
- [23] M. Kozlov, E. Levin, V. Khachatryan and J. Miller, *The BFKL Pomeron calculus in zero transverse dimensions: diffractive processes and survival probability for central diffractive production*, *Nucl. Phys. A* **791** (2007) 382 [[hep-ph/0610084](#)].
- [24] E. Levin and A. Prygarin, *The BFKL Pomeron calculus in zero transverse dimensions: summation of Pomeron loops and generating functional for the multiparticle production processes*, [hep-ph/0701178](#).
- [25] ZEUS collaboration, J. Breitweg et al., *Measurement of the proton structure function F_2 at very low Q^2 at HERA*, *Phys. Lett. B* **487** (2000) 53 [[hep-ex/0005018](#)];
ZEUS collaboration, S. Chekanov et al., *Measurement of the neutral current cross section and F_2 structure function for deep inelastic $e^+ p$ scattering at HERA*, *Eur. Phys. J. C* **21** (2001) 443 [[hep-ex/0105090](#)];
H1 collaboration, C. Adloff et al., *Deep-inelastic inclusive $e p$ scattering at low x and a determination of α_s* , *Eur. Phys. J. C* **21** (2001) 33 [[hep-ex/0012053](#)].
- [26] K. Golec-Biernat and M. Wusthoff, *Saturation effects in deep inelastic scattering at low Q^2 and its implications on diffraction*, *Phys. Rev. D* **59** (1999) 014017 [[hep-ph/9807513](#)].
- [27] F. Gelis, R. Peschanski, G. Soyez and L. Schoeffel, *Systematics of geometric scaling*, *Phys. Lett. B* **647** (2007) 376 [[hep-ph/0610435](#)].
- [28] G. Soyez, *Fluctuations effects in high-energy evolution of QCD*, *Phys. Rev. D* **72** (2005) 016007 [[hep-ph/0504129](#)].
- [29] E. Levin and K. Tuchin, *Solution to the evolution equation for high parton density QCD*, *Nucl. Phys. B* **573** (2000) 833 [[hep-ph/9908317](#)].
- [30] Y. Hatta, E. Iancu, C. Marquet, G. Soyez and D.N. Triantafyllopoulos, *Diffusive scaling and the high-energy limit of deep inelastic scattering in QCD at large- N_c* , *Nucl. Phys. A* **773** (2006) 95 [[hep-ph/0601150](#)].
- [31] C. Marquet, G. Soyez and B.-W. Xiao, *On the probability distribution of the stochastic saturation scale in QCD*, *Phys. Lett. B* **639** (2006) 635 [[hep-ph/0606233](#)].
- [32] E. Brunet, B. Derrida, A.H. Mueller and S. Munier, *A phenomenological theory giving the full statistics of the position of fluctuating pulled fronts*, *Phys. Rev. E* **73** (2006) 056126 [[cond-mat/0512021](#)].
- [33] C. Ewerz and O. Nachtmann, *Towards a nonperturbative foundation of the dipole picture: II. high energy limit*, *Ann. Phys. (NY)* **322** (2007) 1670 [[hep-ph/0604087](#)].